Physics 211C: Solid State Physics

Instructor: Prof. Tarun Grover — Lecture 7

Topic: Fermi Liquid Theory

More on RG for fermi liquids, Anderson Impurity and Kondo problem

Broad idea of RG:

1d Ising model

$$\mathcal{H} = -J \sum s_i \, s_{i+1}$$

$$= \sum_{\{s_i\}} e^{\beta J \sum s_i s_{i+1}}$$

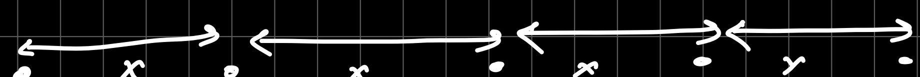
RG = coarse grain, rescaling

$\int$ high energy modes $\rightarrow$ how does it change int b/w low energy modes

$i=1$ $\quad i=2$ $\quad i=3$ $\quad i=4$ $\quad$ - - - -

$$Z = e^{\beta \sum s_{2i} (s_{2i+1} + s_{2i-1})}$$

Trace out even numbered sites



$$\therefore \; Z = \prod_i 2\cosh\left[\beta\left(s_{2i-1} + s_{2i+1}\right)\right]$$

$$= \prod_i A\, e^{\beta' s_{2i-1} s_{2i+1}}$$

$$2\cosh\left(\beta\left(s_{2i-1} + s_{2i+1}\right)\right) = A\, e^{\beta' s_{2i-1} s_{2i+1}}$$

solve for A & β':

$$\boxed{e^{2\beta'} = \cosh(2\beta)}$$

$$e^{2\beta''} = \cosh(2\beta')$$

$$\vdots$$

our RG flow

One can check that as flow proceeds, (β^{-1}) keeps increasing.

$\because \beta^{-1} \propto T$, this implies $T \uparrow$ as flow proceeds.

$$T=0 \;\bullet\!\longrightarrow\!\bullet\; T=\infty$$

$$\Downarrow$$

There must be no SSB at any finite Temp for this model, as it flows to $T=\infty$ fixed point.

In terms of a diff. eqn,

$$v = \tanh\beta$$

$$\tanh\beta' = [\tanh\beta]^{2}$$ → our level of coarse graining

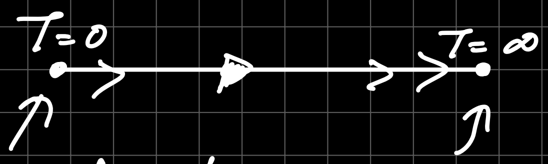
for arbitrary coarse graining

$$\tanh\beta' = [\tanh\beta]^{b} \quad \text{with } b > 1,\; b = e^{dL}$$

for some dL

this leads to $\boxed{\frac{dT}{dl} = \frac{T^2}{2}}$ $\rightarrow$ $d\left(\frac{1}{T}\right) = -\frac{dl}{2}$

$T \sim e^{-\frac{l}{2}}$



unstable phase of matter

stable fixed point or stable phase of matter

\# $\langle s_0 s_r \rangle = \dfrac{Tr\, e^{\beta \sum s_i s_{i+1}} s_0 s_r}{Tr\, e^{\beta \sum s_i s_{i+1}}} \sim e^{-r/\xi}$ ($\because$ no phase Transition)

$\underbrace{\xi \sim e^{2/T}}$

$\Downarrow$

This has relation to BCS theory

$\Delta_k \sim e^{-\frac{1}{g\nu}}$ & ξ_{sc}

& similar result also works in Kondo problem.

Extracting ξ from RG

$\xi(T(l)) = \xi(T)\, e^{-l}$ $\longrightarrow$ comes because we e^{dl} worth of coarse graining at each step

$\because \xi(T) \sim a e^{l}$

$\Rightarrow -\frac{1}{T(l)} + \frac{1}{T} = \frac{l}{2} \Rightarrow \frac{2}{T} = \log\left(\frac{\xi}{a}\right) \Rightarrow \xi(T) \sim a e^{2/T}$

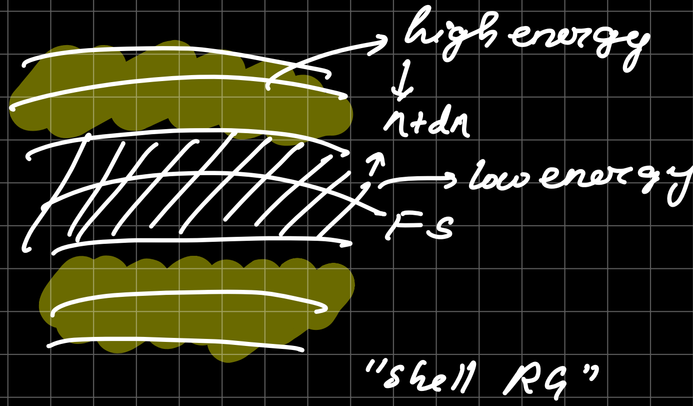
* e.g. of such diverging RG flows:-

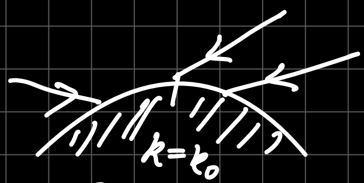
① 1d Ising
② BCS instability
③ Kondo problem
④ QCD

} called "Asymptotically free theories"

RG for fermi liquid

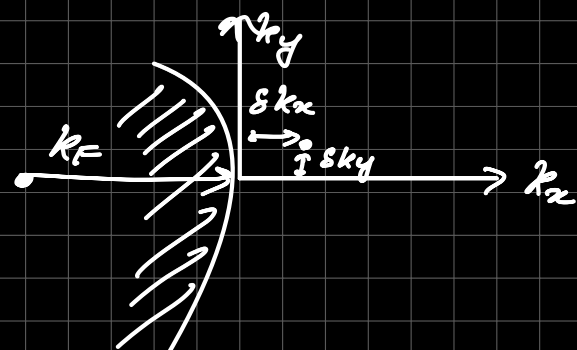
2 ways to do RG





Patch constructⁿ (doesn't account for forward scattering)

Patch RG:



$$\xi = \frac{k^2}{2m} - \frac{k_F^2}{2m}$$

$$= \frac{\left[(k_F + \delta k_x)\hat{x} + \delta k_y \hat{y}\right]^2}{2m} - \frac{k_F^2}{2m}$$

$$= v_F\, \delta k_x + \underbrace{\frac{(\delta k_y)^2}{2m}}_{\text{gets curvature of FS}}$$

$$\therefore \mathcal{H}_0 \approx \psi_k^\dagger \left[v_F k_x + \frac{k_y^2}{2m} \right] \psi_k \qquad \left[\begin{array}{c} \text{COV} \\ \delta k_x \to k_x \\ \delta k_y \to k_y \end{array} \right]$$

1st we look at RG of this free theory

$$S_0 = \int d\tau\, dk_x\, d^{d-1}k_y\; \psi^* \left[\partial_\tau + v_F k_x + \frac{k_y^2}{2m} \right] \psi$$

$$= \int d\tau\, dx\, d^{d-1}y\; \psi^* \left[\partial_\tau - i v_F \partial_x - \frac{\partial_y^2}{2m} \right] \psi$$

Rescale $\quad \tau' = \frac{\tau}{b^2} \quad x' = \frac{x}{b^2} \quad y' = \frac{y}{b} \quad \& \quad \psi' = \psi\, b^{\frac{(d+1)}{2}}$

$$= \int d\tau'\, dx'\, d^{d-1}y'\; \psi^{*\prime}\left[\partial_{\tau'} - i v_F \partial_{x'} - \frac{\partial_{y'}^2}{2m}\right]\psi'$$

Interaction:

$$U \int d\tau\, dx\, d^{d-1}y\;\; \bar\psi_\downarrow(x)\,\bar\psi_\uparrow(x)\,\psi_\uparrow(x)\,\psi_\downarrow(x)$$

$$= U \int d\tau'\, dx'\, d^{d-1}y'\;\; \bar\psi'\bar\psi'\psi'\psi' \quad b^{d+3}\, b^{-2(d+1)}$$

$$= \underbrace{U\, b^{-d+1}}_{U'} \int \;\;"\;\;\;"\;\;\;"\;\;\;"\;\;\;"$$

$$U' = U\, b^{1-d}$$

$\because d > 1$

$\Rightarrow U' < U \Rightarrow$

or for $b = e^{dl}$

$$\frac{dU}{dl} = (1-d)\,U$$

(leading term in RG)
This is at "tree level"

correlat^n fn of

[Gen. a gapless system is unstable
⇓
so many excitations, so there can be some instability]

BCS doubt

If one integrates all high. en. modes,

$$\frac{dU}{dl} = (1-d)\,U + O(U^2)$$

additional corrections

can lead to additional fixed pnts

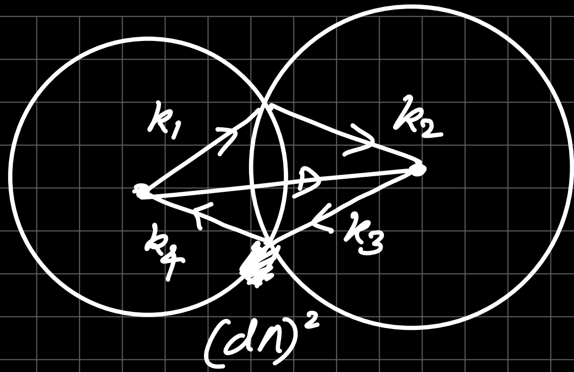
$U=0$ $\quad U^*$

<u>Note</u>:- Patch construction doesn't take into account int. b/w diff. patches

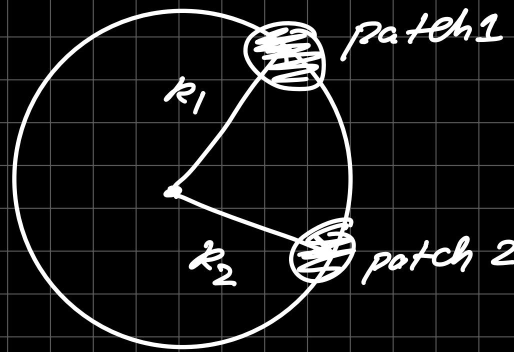
*

$k_1^{\parallel} \simeq k_3^{\parallel}$

$k_4^{\parallel} \simeq k_2^{\parallel}$



$k_1 + k_2 \to k_3 + k_4$



Ref. Nayak's notes for patch construction

$$\int d\omega\, dk_y\, dk_x\; \psi^\dagger \left[i\omega - v_F k_x - \frac{k_y^2}{2m} \right] \psi$$

$$+ \int \underbrace{d\omega_1\, d\omega_2\, d\omega_3}_{\text{energy term}}\, d^2k_1\, d^2k_2\, d^2k_3 \;\; \underbrace{U(k_1, k_2, k_3)}_{f(\theta_1, \theta_2, \theta_3)}$$

$$\psi^\dagger(k_4, \omega_4)\, \psi^\dagger(k_3, \omega_3)\, \psi(k_2, \omega_2)\, \psi(k_1, \omega_1)$$

$$\delta(\underbrace{k_{1y} - k_{3y}}_{})$$

$\Downarrow$

$\because$ $\parallel$ to FS components are equal

$\equiv$ marginal (Check)

$\delta(\alpha x) = \frac{\delta(x)}{|\alpha|}$

$b^6 \; b^{-6}$

$b^3 b^3 b^3$

b

$\omega' \to \frac{\omega}{b}$

$k_x' \to \frac{k_x}{b}$

$k_y' \to \frac{k_y}{\sqrt{b}}$

$\frac{\delta}{b^2}$

$\psi \to \psi' \, b^{-\frac{7}{4}}$

$(b^2 \, b \, \cancel{b^{1/2}}) (\cancel{b^{-1/2}})$

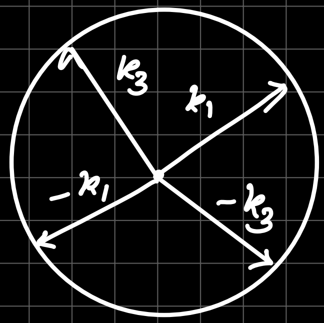
$b^3 \;\; \psi^2$

$b^{-3/2}$

$$b^3 \, b^3 \, b^{3/2} \, b^{-1/2} \, b^{-7} = \underline{(b^0)}$$

$$\underline{\underline{b^{1/2}}}$$

Due to above arguments, $\psi^\dagger \psi^\dagger \psi \psi$ type scattering are irrelevant.



BCS type scattering

$$V\, \psi^\dagger_{k_1\uparrow}\, \psi^\dagger_{-k_1\downarrow}\, \psi_{k_3\uparrow}\, \psi_{-k_3\downarrow}$$

$$\frac{dV}{dl} = V^2 N(0) \quad (\text{"Marginally relevant"})$$

$$\xi \sim e^{\frac{1}{N(0)}}$$

$= \text{Cooper pair size}$

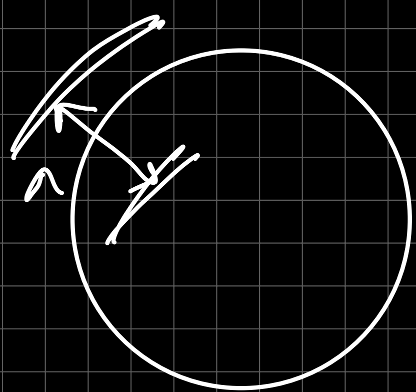
$$\xi_{bcs} \sim \frac{v_F}{\Delta_{BCS}} \qquad \Delta_{BCS} \sim \omega_D\, e^{-\frac{1}{N(0)}}$$

$$T_c \sim \Delta_{BCS}$$

} matches with Ising model type intuition

Shell method of RG

(Shankar's method)



interaction:- BCS & forward scattering

$$\int d^dk\, d\omega\ \psi^\dagger[-i\omega - \xi_k]\psi$$

$$\xi_k = \frac{k^2}{2m} - \mu = v_F\, l$$

$$l = k - k_F$$

$\|k\|$ or polar coordinates

$$d^{d-1}k \sim \underbrace{k^{d-1}}_{k_F^{d-1}}\ \underbrace{dk}_{dl}$$

$$H_0 \approx \int dl\, d\omega\, \psi^\dagger\, [-i\omega - v_F l]\, \psi$$

$$\omega' = \frac{\omega}{b} \qquad l' = \frac{l}{b} \qquad \psi' = \psi\, b^{-3/2}$$

$$\int \underbrace{d\omega_1\, d\omega_2\, d\omega_3}_{b^3}\ \underbrace{dl_1\, dl_2\, dl_3}_{b^3}\ F(\theta_1-\theta_2)\ \underbrace{\psi^\dagger_{k_1}\psi^\dagger_{k_2}\psi_{k_3}\psi_{k_4}}_{b^{-3/2\times 4}}$$

$$= b^0 \quad \text{("Marginally relevant")}$$

$\Downarrow$

One needs to do high energy modes,

$$dF \sim (d\Lambda)^2$$

$$\frac{dF}{d\Lambda} = 0 \quad \text{(even at one loop order)}$$

while $\quad \dfrac{dV}{dL} = V^2$

Ref:- Nayak's notes, Shankar's paper, Polchinski's paper.

<u>Kondo</u> <u>problem</u>: Piers Coleman's notes: Section 1 & 2

Patrick Lee's notes (MIT: 8.512)

(arXiv 0206003)

* $\mathcal{F}$ is marginal $\Rightarrow$ low energy theory is Landau like

$\Downarrow$

"<u>Landau</u> <u>low energy</u> <u>theory</u>"

* For each k pt $\Rightarrow$ $\exists$ a conserved n_k

* exactly marginal $\Rightarrow$ All loops thing in Shankar's thing
* <u>BCS</u> <u>like</u> <u>instability</u> $\Rightarrow$?
* more on FS as an <u>adiabatic</u> <u>evolution</u>.

* CFTs → relevant
  ↳ gapless

scaling dim, corr. length,
Boson mass

Bose liquids

correlatn fn ⇐ Bose liquids
⇓
Bosonic FS

said something abt gapless systems, correlatn length & scaling dim.

irrelevancy of interaction